Summary of Continuous Random Variables

Uniform Random Variable

$S_X = [a, b]$

$$f_X(x) = \frac{1}{b-a} \quad a \le x \le b$$

$$E[X] = \frac{a+b}{2} \quad \text{VAR}[X] = \frac{(b-a)^2}{12} \quad \Phi_X(\omega) = \frac{e^{j\omega b} - e^{j\omega a}}{j\omega(b-a)}$$

Exponential Random Variable

$S_X = [0, \infty)$

$$f_X(x) = \lambda e^{-\lambda x} \quad x \ge 0 \text{ and } \lambda > 0$$

$$E[X] = \frac{1}{\lambda} \quad \text{VAR}[X] = \frac{1}{\lambda^2} \quad \Phi_X(\omega) = \frac{\lambda}{\lambda - j\omega}$$

Remarks: The exponential random variable is the only continuous random variable with the memoryless property.

Gaussian (Normal) Random Variable

$S_X = (-\infty, +\infty)$

$$f_X(x) = \frac{e^{-(x-m)^2/2\sigma^2}}{\sqrt{2\pi}\sigma} \quad -\infty < x < +\infty \text{ and } \sigma > 0$$

$$E[X] = m \quad \text{VAR}[X] = \sigma^2 \quad \Phi_X(\omega) = e^{jm\omega - \sigma^2\omega^2/2}$$

Remarks: Under a wide range of conditions X can be used to approximate the sum of a large number of independent random variables.

Gamma Random Variable

$S_X = (0, +\infty)$

$$f_X(x) = \frac{\lambda(\lambda x)^{\alpha-1}e^{-\lambda x}}{\Gamma(\alpha)} \quad x > 0 \text{ and } \alpha > 0, \lambda > 0$$

where $\Gamma(z)$ is the gamma function.

$$E[X] = \alpha/\lambda \quad \text{VAR}[X] = \alpha/\lambda^2 \quad \Phi_X(\omega) = \frac{1}{(1 - j\omega/\lambda)^\alpha}$$

Special Cases of Gamma Random Variable:

– m-Erlang Random Variable: $\alpha = m$, a positive integer

$$f_X(x) = \frac{\lambda e^{-\lambda x}(\lambda x)^{m-2}}{(m-1)!} \quad x > 0 \quad \Phi_X(\omega) = \left(\frac{1}{1 - j\omega/\lambda}\right)^m$$

Remarks: An m-Erlang random variable is obtained by adding m independent exponentially distributed random variables with parameter λ.

– Chi-Square Random Variable with k degrees of freedom: $\alpha = k/2, k$ a positive integer, and $\lambda = 1/2$

$$f_X(x) = \frac{x^{(k-2)/2}e^{-x/2}}{2^{k/2}\Gamma(k/2)} \quad x > 0 \quad \Phi_X(\omega) = \left(\frac{1}{1 - 2j\omega}\right)^{k/2}$$

Remarks: The sum of k mutually independent, squared zero-mean, unit-variance Gaussian random variables is a chi-square random variable with k degrees of freedom.

Laplacian Random Variable

$S_X = (-\infty, \infty)$

$f_X(x) = \frac{\alpha}{2} e^{-\alpha|x|} \quad -\infty < x < +\infty$ and $\alpha > 0$

$E[X] = 0 \quad \text{VAR}[X] = 2/\alpha^2 \quad \Phi_X(\omega) = \frac{\alpha^2}{\omega^2 + \alpha^2}$

Rayleigh Random Variable

$S_X = [0, \infty)$

$f_X(x) = \frac{x}{\alpha^2} e^{-x^2/2\alpha^2} \quad x \geq 0$ and $\alpha > 0$

$E[X] = \alpha\sqrt{\pi/2} \quad \text{VAR}[X] = (2 - \pi/2)\alpha^2$

Cauchy Random Variable

$S_X = (-\infty, +\infty)$

$f_X(x) = \frac{\alpha/\pi}{x^2 + \alpha^2} \quad -\infty < x < +\infty$ and $\alpha > 0$

Mean and variance do not exist. $\quad \Phi_X(\omega) = e^{-\alpha|\omega|}$

Pareto Random Variable

$S_X = [x_m, \infty) \; x_m > 0$

$$f_X(x) = \begin{cases} 0 & x < x_m \\ \alpha \frac{x_m^\alpha}{x^{\alpha+1}} & x \geq x_m \end{cases}$$

$E[X] = \frac{\alpha x_m}{\alpha - 1}$ for $\alpha > 1 \quad \text{VAR}[X] = \frac{\alpha x_m^2}{(\alpha-2)(\alpha-1)^2}$ for $\alpha > 2$

Remarks: The Pareto random variable is the most prominent example of random variables with "long tails," and can be viewed as a continuous version of the Zipf discrete random variable.