Summary of Discrete Random Variables

Bernoulli Random Variable

$S_X = \{0, 1\}$

$p_0 = q = 1 - p \quad p_1 = p \quad 0 \leq p \leq 1$

$E[X] = p \quad \text{VAR}[X] = p(1-p) \quad G_X(z) = (q + pz)$

Remarks: The Bernoulli random variable is the value of the indicator function I_A for some event A; $X = 1$ if A occurs and 0 otherwise.

Binomial Random Variable

$S_X = \{0, 1, \ldots, n\}$

$p_k = \binom{n}{k} p^k (1-p)^{n-k} \quad k = 0, 1, \ldots, n$

$E[X] = np \quad \text{VAR}[X] = np(1-p) \quad G_X(z) = (q + pz)^n$

Remarks: X is the number of successes in n Bernoulli trials and hence the sum of n independent, identically distributed Bernoulli random variables.

Geometric Random Variable First Version:

$S_X = \{0, 1, 2, \ldots\}$

$p_k = p(1-p)^k \quad k = 0, 1, \ldots$

$E[X] = \dfrac{1-p}{p} \quad \text{VAR}[X] = \dfrac{1-p}{p^2} \quad G_X(z) = \dfrac{p}{1 - qz}$

Remarks: X is the number of failures before the first success in a sequence of independent Bernoulli trials. The geometric random variable is the only discrete random variable with the memoryless property.

Second Version:

$S_{X'} = \{1, 2, \ldots\}$

$p_k = p(1-p)^{k-1} \quad k = 1, 2, \ldots$

$E[X'] = \dfrac{1}{p} \quad \text{VAR}[X'] = \dfrac{1-p}{p^2} \quad G_{X'}(z) = \dfrac{pz}{1 - qz}$

Remarks: $X' = X + 1$ is the number of trials until the first success in a sequence of independent Bernoulli trials.

Negative Binomial Random Variable

$S_X = \{r, r+1, \ldots\}$ where r is a positive integer

$p_k = \binom{k-1}{r-1} p^r (1-p)^{k-r} \quad k = r, r+1, \ldots$

$E[X] = \dfrac{r}{p} \quad \text{VAR}[X] = \dfrac{r(1-p)}{p^2} \quad G_X(z) = \left(\dfrac{pz}{1 - qz}\right)^r$

Remarks: X is the number of trials until the r th success in a sequence of independent Bernoulli trials.

Poisson Random Variable

$S_X = \{0, 1, 2, \ldots\}$

$p_k = \frac{\alpha^k}{k!}e^{-\alpha} \quad k = 0, 1, \ldots \qquad \text{and } \alpha > 0$

$E[X] = \alpha \quad \text{VAR}[X] = \alpha \quad G_X(z) = e^{\alpha(z-1)}$

Remarks: X is the number of events that occur in one time unit when the time between events is exponentially distributed with mean $1/\alpha$.

Uniform Random Variable

$S_X = \{1, 2, \ldots, L\}$

$p_k = \frac{1}{L} \quad k = 1, 2, \ldots, L$

$E[X] = \frac{L+1}{2} \quad \text{VAR}[X] = \frac{L^2-1}{12} \quad G_X(z) = \frac{z}{L}\frac{1-z^L}{1-z}$

Remarks: The uniform random variable occurs whenever outcomes are equally likely. It plays a key role in the generation of random numbers.

Zipf Random Variable

$S_X = \{1, 2, \ldots, L\}$ where L is a positive integer

$p_k = \frac{1}{c_L}\frac{1}{k} \quad k = 1, 2, \ldots, L$ where c_L is given by Eq. (**??**)

$E[X] = \frac{L}{c_L} \quad \text{VAR}[X] = \frac{L(L+1)}{2c_L} - \frac{L^2}{c_L^2}$

Remarks: The Zipf random variable has the property that a few outcomes occur frequently but most outcomes occur rarely.